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MALQUIST PRODUCTIVITY INDEX: IDEA, FRAMEWORK, and ITS EXTENSION ON PARAMETRIC APPROACH

Suyanto

ABSTRACT

This paper explores the theoretical approach of Malmquist Productivity Index (MPI). The basic idea of Stan Malmquist on the geometrical quantity index is discussed. The development of MPI framework for the output distance function is then explored. The extension of MPI on the stochastic production frontier is investigated further, to show the current development of MPI in parametric approach.

Keywords: *Malmquist Productivity Index, Parametric Approach, Non-parametric*

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The theoretical idea of the MPI was first introduced by Caves *et al.* (1982), based on the quantity index of Sten Malmquist (1953). In their paper, Caves *et al.* state that the *Malmquist* Index is a ratio between two distance functions that compares a firm's productivity with that of an alternative firm. Following this theoretical idea, the MPI is then used widely to measure productivity change in both parametric and non-parametric techniques of empirical studies. The commonly used non-parametric technique for decomposing the MPI is the Data Envelopment Analysis (DEA) and the usual parametric techniques is the Stochastic Frontier Approach (SFA).

Färe *et al.* (1994) author the pioneering paper showing that the MPI can be empirically implemented by means of DEA techniques. Based on the idea of Nishimizu and Page (1982), and in the spirit of Farrell's (1957) efficiency context, Färe *et al.* (1994) proposes an initial decomposition of productivity growth into technological change and efficiency change. Extending Färe *et al.* (1994), the more recent studies on MPI using DEA techniques relax the constant return to scale (CRS) assumption for separating scale efficiency change from technical efficiency change. These recent studies include Grifell-Tatje and Lovell (1995), Fare and Grosskopf (1996), Lovell (2003) and Zofio (2007).

In a related method but with a different approach, Fuentes *et al.* (2001) provide a framework of the MPI using the SFA technique. They show that distance functions from SFAs can be used for decomposing MPI into technological change and efficiency change. Extending the decomposition, Balk (2001) and Orea (2002) include scale efficiency change as a component of productivity change for capturing economies of scale. In his model, Balk (2001) shows that the scale efficiency change can be derived directly from an output-oriented translog distance function by applying Ray's (1998) procedure. On the other hand, Orea (2002) derives the scale efficiency change by adopting Diewert's (1976) quadratic lemma. Both studies show that the measure of MPI with a scale efficiency change is appropriate for producers under variable return to scale (VRS). To show the decomposition in an empirical context, Balk applies his model to Dutch rubber-processing firms and Orea applies his model to Spanish savings banks.

Both the parametric and the non-parametric decompositions of MPI are defined in terms of Shephard's (1970) distance function. This distance function can be generalized from either an input-oriented or an output-oriented objective. From the input orientation, the distance function is defined as the minimum feasible contraction of the input vector with the output vector held fixed (*i.e.*, the input minimization objective). Likewise, the output distance function is defined as the maximum feasible expansion of the output vector given a fixed input vector (*i.e.*, the output maximization objective). In this paper, the output-oriented Shephard's distance function is adopted in order to focus on output productivity.

The following section provides a brief formal discussion of the output-oriented distance function before proceeding further on the *Malmquist* productivity index (MPI). An original decomposition of MPI and its development are presented in the second section. The parametric decomposition of MPI proposed by Orea (2002), which is the chosen MPI in this paper, is discussed in the third section. The last section concludes this paper.

II. An Output-Oriented Shephard's (1970) Distance Function

Consider a panel of *i* (*i=1,...,N*) producers observed in *t* (*t=1,...,T*) periods, transforming input vectors $x_i^t = (x_{1i}^t,...,x_{ni}^t) \in \Re_+^n$ into output vectors $y_i^t = (y_{1i}^t,...,y_{mi}^t) \in \Re_+^m$. Given this information, technology can be represented by the production possibility set of feasible input-output combinations

$$S^t = \left\{ \left(x^t, y^t \right); x^t \in \Re_+^n \text{ can produce } y^t \in \Re_+^m \right\}, \ t = 1,...,T \tag{1}$$

which are assumed to satisfy the usual regularity axioms of production theory (for example, Fare and Primont, 1995). Under this framework, a valid representation of the technology from the *i*-th firm perspective using the output oriented Shephard's (1970) distance function, $D_O^t(x_i^t, y_i^t): \Re_+^n \times \Re_+^m \to \Re_+^1 \cup \{\infty\}$, is defined as

$$D_O^t\left(x_i^t, y_i^t\right) \equiv \inf_\theta \left\{\theta > 0 : \left(x_i^t, y_i^t / \theta\right) \in S^t \right\} \qquad (2)^1$$

The technology in equation (2) is assumed linearly homogenous of degree +1 in *y* and non-increasing in *x*. For any period of time *t*, a complete characteristisation of the technology of firm *i*, is expressed as

$$D_O^t\left(x_i^t, y_i^t\right) \le 1 \quad \Leftrightarrow \mathrm{y}_\mathrm{i}^\mathrm{t} \in S^t \tag{3}$$

Equation (3) serves as a criterion for measuring the relative distance from the frontier of the technology set to any point of input-output combination inside the set. Following an output distance function introduced by Shephard (1970)[2], the maximum feasible expansion of the output vector with the input vector held fixed is $D_O^t(x_i^t, y_i^t) = 1$. In this condition, the evaluated firm is said to be efficient belonging to the best-practice technology, which is represented by the subset isoquant $S^t(x, y) = \left\{ (x, y) : D_O^t(x_i^t, y_i^t) = 1 \right\}$. In contrast, if $D_O^t(x_i^t, y_i^t) < 1$, a radial expansion of the output vector y_i^t is feasible within the production technology for the observed input level x_i^t and the evaluated firm is said to be inefficient.

To allow for non-negativity and a point outside of the technology set (which shows a corresponding technology in a different period), the technology frontier of S^t, for $x_i^t \in \Re_+^n$, can be defined as

$$\partial S^t = \left\{ (x, y) : y \in S^t, \lambda y \notin S^t, \forall \lambda \in (1, +\infty) \right\}, y \in \Re_+^m \tag{4}$$

[1] The symbol of *inf* denotes "infimum" or "the greatest lower bound".

[2] The Shephard's output distance function measures the relative distance from the outer boundary of S^k to any point inside this set using a radial expansion of the output vector y_i^t along the ray from the origin in $\Re_+^M$, while keeping the input fixed.

From equation (4), a non-zero point inside the output set but not in the frontier, *i.e.*, $y^t \in S^t$ for $y^t \neq 0$, would be result in $0 < D_O^t(x_i^t, y_i^t) < 1$. This point shows the existence of the Farrell's (1957) technical inefficiency for the related firm. In contrast, a point outside the frontier that represents a corresponding technology in a different period, *i.e.*, $y^t \notin S^t$, $\exists \lambda > 0$: $\lambda y^t \in S^t$, would be expressed as $D_O^t(x_i^t, y_i^t) > 1$. Furthermore, the point on the technology frontier is shown by a distance function equal to one, or expressed as $D_O^t(x_i^t, y_i^t) = 1 \Leftrightarrow y_i^1 \in \partial S^t$. Lastly, the point in the origin representing the distance function equals zero, which occurs if and only if the output equals zero, or expressed formally as $D_O^t(x_i^t, y_i^t) = 0 \Leftrightarrow y_i^t = 0$. In a practical sense, this zero distance function is unlikely to occur since a producer may not produce at zero output given an availability of inputs. Hence, empirical studies, such as the one conducted in this thesis, hardly consider the zero distance function.

III. The Original Decomposition of MPI

The decomposition of MPI was originated by Caves, Christensen, and Diewert (1982). Using the output-oriented Shephard's (1970) distance function, as described above, Caves *et al.* (1982) shows that a change in a firm's technology frontier between two consecutive periods can be decomposed into two components (technological change and efficiency change).

Suppose that firm *i*'s technology is observed in two periods, *t* = *1*, *2*. The technology for these two periods is represented by $\left(x_i^1, y_i^1\right)$ and $\left(x_i^2, y_i^2\right)$, respectively. The output-oriented MPI as introduced by Caves *et al.* (1982) can be defined as:

$$M_O^{12}\left(x_i^1, y_i^1, x_i^2, y_i^2\right) = \left(\frac{D_O^1\left(x_i^2, y_i^2\right)}{D_O^1\left(x_i^1, y_i^1\right)} \times \frac{D_O^2\left(x_i^2, y_i^2\right)}{D_O^2\left(x_i^1, y_i^1\right)}\right)^{\frac{1}{2}} \tag{5}$$

where $M_O^{12}\left(x_i^1, y_i^1, x_i^2, y_i^2\right)$ is a MPI for period *t=1,2* , $D_O^1\left(x_i^2, y_i^2\right)$ represents a distance function that compares second period firms to first period technology, $D_O^1\left(x_i^1, y_i^1\right)$ is a distance function for firm *i* at the first period technology, $D_O^2\left(x_i^2, y_i^2\right)$ denotes a distance function for firm *i* at the second period technology, and $D_O^2\left(x_i^1, y_i^1\right)$ is a distance function that compares first period firms to the second period technology.

The right-hand side of equation (5) can be rewritten as:

$$M_O^{12}\left(x_i^1, y_i^1, x_i^2, y_i^2\right) = \left(\frac{D_O^1\left(x_i^1, y_i^1\right)}{D_O^2\left(x_i^1, y_i^1\right)} \times \frac{D_O^1\left(x_i^2, y_i^2\right)}{D_O^2\left(x_i^2, y_i^2\right)}\right)^{\frac{1}{2}} \times \frac{D_O^2\left(x_i^2, y_i^2\right)}{D_O^1\left(x_i^1, y_i^1\right)} \tag{6}$$

$$=TC_O^{1,2}\left(x_i^1, y_i^1, x_i^2, y_i^2\right)\times TEC_O^{1,2}\left(x_i^1, y_i^1, x_i^2, y_i^2\right)$$

where *TC* is technological change (*i.e.*, the shift in the technology frontier between the two periods) and *TEC* is technical efficiency change (*i.e.*, the movement to the technology frontier).

After the original decomposition of MPI, as in equation (6), researchers then develop various possible decompositions of MPI by taking into account the scale efficiency change. The rationale behind including the scale efficiency is to relax the constant return to scale assumption. Grifell-Tatje and Lovell (1995) argue that a decomposition of MPI without taking into account scale efficiency may not measure productivity change as the change in average productivity. Similarly, Ray and Desli (1997) show that the efficiency change components in MPI consist not only of technical efficiency change but also scale efficiency change if firms operate under variable returns to scale. In a more formal argument, Färe *et al.* (1994) and Grifell-Tatje and Lovell (1997) indicate that the constant returns to scale assumption in the original MPI is not properly applied for firms under a competitive environment. They suggest generalizing the MPI with a scale component that takes into account the contribution of returns to scale.

In recent years, various possible ways have been proposed to develop a generalized MPI (*i.e.*, a measure of productivity change attributable to scale economies). The non-parametric techniques (DEA) for decomposing a generalized MPI are proposed by Fare *et al.* (1994) and Grifell-Tatje and Lovell (1997), and the parametric (SFA) decompositions are addressed by Balk (2001) and Orea (2002). In a non-parametric context, the scale efficiency change is measured by comparing the scale efficiency level between two periods. The level of scale efficiency is calculated using the ratio of distance function values corresponding to constant and variable returns to scale technology. In a parametric context, the scale efficiency change is directly measured from the output-oriented *translog* distance function with variable returns to scale.

Both parametric and non-parametric techniques have their own merits and demerits in decomposing a generalized MPI. The debate over which one is the more appropriate technique continues. This thesis adopts a parametric technique for a consistency with the stochastic frontier approach (SFA) employed in the previous section. From the estimates of SFA and the technical efficiency scores in the previous section, a generalized MPI and its components can be calculated. The next sub-section explains the parametric decomposition of a generalized MPI as proposed by Orea (2002).

IV. Parametric Decomposition of MPI with Scale Efficiency Change

Under a parametric decomposition of a generalized MPI, the distance function is represented by a specific functional form. Suppose that firm *i*'s technology in time *t* is represented by a transcendental logarithmic (*translog*) output-oriented distance function, $\ln D_O\left(y_{it}, x_{it}, t\right)$. By applying Diewert's (1976) Quadratic Identity Lemma, Orea (2002) shows that the logarithm of a generalized output-oriented MPI between time period *t* and *t+1*, $G_O^{t,t+1}$, can be decomposed into technical efficiency change (TEC), technological change (TC), and scale efficiency change (SEC), as expressed below:

$$G_{Oi}^{t,t+1} = TEC_i^{t,t+1} + TC_i^{t,t+1} + SEC_i^{t,t+1} \qquad (7)$$

where

$$TEC_i^{t,t+1} = \ln D_O(y_{i,t+1}, x_{i,t+1}, t+1) - \ln D_O(y_{it}, x_{it}, t) \tag{8}$$

$$TC_i^{t,t+1} = -\frac{1}{2}\left[\frac{\partial \ln D_O(y_{i,t+1}, x_{i,t+1}, q)}{\partial(t+1)} + \frac{\partial \ln D_O(y_{it}, x_{it}, t)}{\partial t}\right] \tag{9}$$

$$SEC_i^{t,t+1} = \frac{1}{2}\sum_{n=1}^{N}\left[\frac{\varepsilon_{i,t+1}-1}{\varepsilon_{i,t+1}}\varepsilon_{i,t+1.n} + \frac{\varepsilon_{it}-1}{\varepsilon_{it}}\varepsilon_{itn}\right].\ln\left[\frac{x_{i,t+1.n}}{x_{itn}}\right] \tag{10}$$

where $\varepsilon_{it} = \sum_{n=1}^{N}\varepsilon_{nit}$ is the scale elasticity such that $\varepsilon_{nit} = \dfrac{\partial \ln D_O(y_{it}, x_{it}, t)}{\partial \ln x_{itn}}$.

For the purpose of this paper, it is necessary to assume that the output is only one.[3] Hence, the econometric version of a stochastic *translog* output-oriented distance function for a firm panel data can be represented by:

$$\ln y_{it} = \beta_0 + \sum_{n=1}^{N}\beta_n \ln x_{nit} + \frac{1}{2}\sum_{n=1}^{N}\sum_{k=1}^{K}\beta_{nk}\ln x_{nit}\ln x_{kit} + \beta_t t + \frac{1}{2}\beta_{tt}t^2 + \sum_{n=1}^{N}\beta_{nt}\ln x_{nit}t + v_{it} - u_{it} \tag{11}$$

where y_{it} represents output of firm i at time period t, x_{nit} represents input n for firm i at time period t, v_{it} is a stochastic error component, and u_{it} represents the error component related to technical inefficiency.

The technology frontier of the distance function (*i.e.*, $D_O(y_{it}, x_{it}, t) = 1$) is expressed as

$$\ln y_{it}^P = \beta_0 + \sum_{n=1}^{N}\beta_n \ln x_{nit} + \frac{1}{2}\sum_{n=1}^{N}\sum_{k=1}^{K}\beta_{nk}\ln x_{nit}\ln x_{kit} + \beta_t t + \frac{1}{2}\beta_{tt}t^2 + \sum_{n=1}^{N}\beta_{nt}\ln x_{nit}t \tag{12}$$

where y_{it}^P represents the potential maximum output that can be achieve given a set of inputs.

Given equations (11) and (12), the distance to the technology frontier can be calculated from

$$\ln D_O(y_{it}, x_{it}, t) = \ln y_{it} - \ln y_{it}^P - v_{it} \tag{13}$$

which is equivalent to technical inefficiency, u_{it}. Following Coelli *et al.* (2005), the technical efficiency change (TEC) from Equation (8) can be measured by:

$$TEC_i^{t,t+1} = \ln TE_{i,t+1} - \ln TE_{it} \tag{14}$$

The technological change (TC) index can be obtained from Equations (9) and (11) as follows:

[3] The original output-oriented translog distance function $\ln D_O(y_{it}, x_{it}, t)$ is expressed in a multi-outputs and multi-inputs function. The complete translog distance function is given in Orea (2002). In this paper, the output is assumed as only one, and the translog distance function for the econometric version is given in equation (11). An assumption of one output in this thesis is related to the availability of data.

$$TC_{i,t+1,t}=\frac{1}{2}\left[\sum_{n=1}^{N}\beta_{tn}\ln x_{i,t+1,n}+\sum_{n=1}^{N}\ln x_{itn}+2\beta_{t}+2\beta_{tt}\left((t+1)+t\right)\right] \tag{15}$$

From Equations (11), the scale elasticity is expressed as

$$\varepsilon_{nit}=\beta_{n}+\frac{1}{2}\sum_{k=1}^{K}\beta_{nk}x_{nit}+\beta_{nt}t \tag{16}$$

The index of scale efficiency change can then be calculated by using Equations (10) and (16).

V. Conclusion

This paper explores the theoretical framework and the development of the Malmquist Productivity Index (MPI). The quantity index of Stan Malmquist is discussed in the beginning of this paper as a basic fundamental for the recent developed MPI. The output-distance function of Shephard (1970) is presented as the framework within the theoretical analysis of MPI. The original decomposition of Caves, Christensen, and Diewert (1982) is then explored under the framework of the output-distance function. This paper presents the parametric decomposition of MPI with scale efficiency change, to show the application of MPI on the productivity measures.

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